Asset Pricing

Module 6, 2024-25

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Course information

- **Course Website:** my.nes.ru
- **Instructor's Office Hours:** after each lecture or by previous appointment
- **TA:** TBA

Course description

This course gives an introduction to the economics and mathematics of financial markets. Two basic pricing principles are considered. The first is the absence of arbitrage and is mostly used for the pricing of derivative assets (e.g. forward or option contracts) relative to basic or underlying assets (e.g. stocks or interest rates).

In order to price the whole universe of financial assets, however, we need to investigate how investors optimally make their investment decisions and how the coordination of these investors on the financial markets leads to the formation of prices. This is the second principle – equilibrium analysis. We start with classical problems of how investors choose optimal portfolios, in particular the mean-variance or Markowitz problem. We then set up portfolio choice problem in a dynamic context. Letting several investors interact leads us to equilibrium asset pricing models, such as the famous capital asset pricing model. We also learn how to solve dynamic equilibrium models. Even though many advanced finance models are formulated in continuous time, in this introductory course we limit ourselves to discrete time models.

The focus of the course is on theory, but we shall comment on some empirical evidence and on how these theories are tested and used in financial practice. It will be useful to absorb the ideas and mathematics of financial models by doing small applications on the computer. R will be the programming languages of choice in class examples and I encourage you to use it to solve homework exercises even though you may use other programming languages.

Course requirements, methodology, grading, and attendance policies

Prerequisites: Microeconomics 1-3, Probability theory, Mathematics for Economists.

Description of course methodology: The course consists of lectures, seminars and homework assignments. Seminars mostly serve to introduce you to R programming, answer questions on pending homework assignments, show the solutions to submitted homework exercises, and to present some additional worked-out examples.

Attendance: Class participation does not directly contribute towards your grade. However, active participation will help you to understand the material better. Also, it saves you time since working with textbooks and lecture notes alone does not tell you the exact choice of topics and material and some tricks to solve certain types of problems. In short, if you want to save time you should come to class.

Grading:

- *Homework assignments* are marked and account for 20% of the final grade. Every homework problem set comes with a due date and time. Late submission up to 24 hours leads to a discount of 20%; submitting even later or not submitting leads to a zero grade for this assignment.
- A short *test* in the middle of the term accounts for 20%.
- The rest of the grade (60%) comes from the *final exam*.
- Exam questions in the midterm and the final exam will be either exercises, similar to homework assignments, or questions concerning interpretation of obtained results, on concepts explained in class, lecture notes or relevant book chapters.

The *grading scale* is

%	Grade
[0, 35)	2
[35, 39)	2,7
[39, 44)	3
[44, 48)	3,3
[48, 53)	3,7
[53, 58)	4
[58, 63)	4,3
[63, 73)	4,7
[73, 80)	5
[80, 100]	5,3

Missing an exam. If a student misses the midterm exam and the NES administration has verified that the reason is legitimate, the final exam will have a weight of 80%.
If a student misses the final exam and the NES administration has verified that the reason is legitimate, the student may take a replacement exam, covering all material in the course.
If a student's grade for the course is failing, the student retains the right to have one make-up (beyond the replacement exam mentioned above). It will be written and comprehensive, covering all material in the course. The degree of difficulty will be no less than in the original exam.

Course contents

This outline lists the topics to be covered in the course and gives an approximate time schedule.

Part 1 -- Asset Pricing Models and the No-Arbitrage Principle

- Week 1. Introduction: The terminology of financial markets; Bond prices and interest rates under certainty. Uncertainty, replicating portfolios, Arrow-Debreu securities.
- Week 2-3. Arbitrage and market completeness. The Fundamental Theorem of Finance. Applications: The pricing of forwards and futures; The Binomial model of option pricing.

Part 2 -- Individual optimality

- Week 3-4. Review of individual preferences, utility theory, and risk-aversion. Optimal consumption and portfolio choice in one period. Mean-variance analysis.

Part 3 -- Equilibrium models

- Week 5-6. Short review of equilibrium fundamentals: Concept of equilibrium, representative agent, existence and Pareto-optimality. Consumption capital asset pricing model and the capital asset pricing model. Multi-factor models. Empirical testing of asset pricing models.
- Week 6-7. Asset pricing in discrete time dynamic models: The Lucas model. Asset pricing puzzles. Rational Asset Price Bubbles.

Sample tasks for course evaluation

1a) Define and draw a graph of the mean-variance frontier. Show the efficient part of the frontier on the graph.
1b) In the mean-variance optimization problem *with a risk-less asset* available, we found the following expression for the optimal weights of each asset within the portfolio of risky assets

$$\tilde{\boldsymbol{\pi}}^* = \frac{\mathbf{V}^{-1}(\boldsymbol{\mu} - r\mathbf{1})}{A - rC}$$

Assuming $r < A/C$, show that this portfolio lies on the *efficient frontier with only risky assets*. Recall that optimal portfolios in the problem with risky assets only are of the form

$$\boldsymbol{\pi}^* = \boldsymbol{g} + \boldsymbol{h}\bar{\mu}$$

where $\bar{\mu}$ is the target portfolio return, $\boldsymbol{g} = \frac{1}{D}[B(\mathbf{V}^{-1}\mathbf{1}) - A(\mathbf{V}^{-1}\boldsymbol{\mu})]$, $\boldsymbol{h} = \frac{1}{D}[C(\mathbf{V}^{-1}\boldsymbol{\mu}) - A(\mathbf{V}^{-1}\mathbf{1})]$, and

$$\begin{aligned} A &= 1^\top \mathbf{V}^{-1}\boldsymbol{\mu} = \mu^\top \mathbf{V}^{-1}\mathbf{1} \\ B &= \mu^\top \mathbf{V}^{-1}\boldsymbol{\mu} \\ C &= 1^\top \mathbf{V}^{-1}\mathbf{1} \\ D &= BC - A^2 \end{aligned}$$

2) Suppose the existence of a return-generating process in the form of a two-factors model without idiosyncratic risk. The information about the expected returns of three assets (A, B and C) and their factor loadings (sensitivities) with respect to the two factors are given in the following table:

	$E[R_j]$	β_{j1}	β_{j2}
A	0.0330	0.9	0.8
B	0.0315	0.7	0.9
C	0.0325	1.0	0.5

a) Obtain the portfolio weights of the two factor portfolios and their expected returns.
b) Obtain also the replicating portfolio for the risk-less asset and its return.
c) If the return of the riskless asset were $r = 0.025$, which arbitrage strategy would you pursue, and what would be the profit of this strategy?
d) Recall that the fundamental pricing equation of the APT reads (in the case of a two-factor model)

$$E[R_j] = \lambda_0 + \lambda_1\beta_{j1} + \lambda_2\beta_{j2}$$

Compute the values of λ_0, λ_1, and λ_2 and confirm that they represent the risk-less return and the risk premia of the two factor portfolios, respectively.

3) St. Petersburg Associates, a firm of financial analysts specializing in Russian financial markets, forecasts that the stock of the Siberian Drilling Company will be worth 1,000 rubles per share one year from today. If the riskless interest rate on Russian government securities is 10% and the

expected return to the market portfolio is 18%. Determine how much you would pay for a share of Siberian Drilling stock today if the beta of Siberian Drilling is 3.

Course material

Required textbooks and materials

- Lecture notes.
- Pennacchi, George, The Theory of Asset Pricing, Pearson Addison Wesley, 2008.

Additional material

- Back, Kerry, Asset Pricing and Portfolio Choice Theory, Oxford University Press, 2nd edition 2017.
- Capiński, Marek and Tomasz Zastawniak, Mathematics for Finance – An Introduction to Financial Engineering, 2nd edition, Springer 2011.
- Cvitanic, Jaksa and Fernando Zapatero, Introduction to the Economics and Mathematics of Financial Markets, MIT Press, 2004.
- Joshi, Mark S. and Jane M. Paterson, Introduction to mathematical portfolio theory, Cambridge University Press.
- LeRoy, Stephen and Jan Werner, Principles of Financial Economics, Cambridge University Press, 2001.

Academic integrity policy

Cheating, plagiarism, and any other violations of academic ethics at NES are not tolerated. Assume that all work is to be done on your own without outside assistance unless otherwise stated. When you use or repeat someone else's idea, you must cite the author or source.

For computer exercises you are permitted to use computer code you find in books or on the internet, but please note the source in your code to give credit to the author or source. You may not use classmate's or classmates' computer code.